The η-Nucleon Scattering Length and Effective Range

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Abstract

The coupled ηN , πN system is described by a K-matrix method. The parameters in this model are adjusted to get an optimal fit to $\pi N \to \pi N$, $\pi N \to \eta N$ and $\gamma N \to \eta N$ data in an energy range of about 100MeV each side of the η threshold.

In the notation $T^{-1} + iq_\eta = 1/a + \frac{r_0}{2} q_\eta^2 + s q_\eta^4$, q_η being the momentum in the ηN center-of-mass, the resulting effective range parameters for ηN scattering are found to be

a(fm) = 0.75(4)–i0.27(3), r_0(fm) = –1.50(13)–i0.24(4)

and s(fm^3) = –0.10(2)–i0.01(1)

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The pion-nucleon and pion-nucleus interactions have been much studied, both theoretically and experimentally, for many years. However, the corresponding interactions of the eta meson, mainly because of the lack of η-beams, have – by comparison – been neglected. The main interest in η's has been the possibility of η-nuclear quasi-bound states. Such states were first predicted by Haider and Liu [1] and Li et al. [2], when it was realised that the η-nucleon interaction was attractive. Calculations by Ueda indicated [3] that this may happen already in the η-deuteron system. If these states exist, then one may expect them to be narrow in few-nucleon systems, and so be easier to detect there. The first verification of this hypothesis was made by Wilkin [4], who has suggested that an indirect effect of such a state is seen in the rapid slope of the $pd \rightarrow\ ^3He\eta$ amplitude detected just above the η production threshold [5]. Also an indication of strong three-body ηpp correlations follows from the measurement of the $pp \rightarrow pp\eta$ cross sections in the threshold region [6].

It has been shown in Ref. [7] that the strengths of ηd interactions, in particular the magnitude of the scattering length in this system and the position of quasi-bound or virtual state are very sensitive to the value of the ηN scattering length. Moreover, the behaviour of the ηN scattering matrix off the energy-shell was found to be important. In order that these η-nucleus studies can be put on a firmer foundation, it is, therefore, necessary that a better parametrization of the basic η-nucleon interaction be available. With this in mind, a three channel analysis of the eta-nucleon(ηN), pion-nucleon(πN) and the two-pion-nucleon($\pi\pi N$) three-body system is carried out. This is done in terms of a K-matrix based on pion-nucleon amplitudes and eta-production cross sections – the actual data being the πN amplitudes of Arndt et al. [8], the $\pi N \rightarrow \eta N$ cross sections

reviewed by Nefkens [9] and the $\gamma p \to \eta p$ data of Krusche et al. [10]. In Ref. [11] it is shown that the photoproduction cross section runs essentially parallel to the electroproduction cross section in the region some 100 MeV above threshold. Therefore, including electroproduction data into the following analysis is not expected to lead to a different conclusion.

This analysis is now carried out and, we believe, it improves analyses done directly in terms of resonant T matrices. The main motivation for this study is to extract the eta-nucleon scattering length *and* effective range and to determine in these quantities the uncertainties allowed by the existing data. The next check and refinement is expected to follow from the few-body η physics.

For s-wave scattering in a system consisting of the two channels πN and ηN – here denoted simply by the indices π and η – the K-matrix, which is essentially a generalisation of the scattering length, and the T-matrix that follows from it, can be written as

$$\hat{K} = \begin{pmatrix} K_{\pi\pi} & K_{\eta\pi} \\ K_{\pi\eta} & K_{\eta\eta} \end{pmatrix} \quad \text{and} \quad T = \begin{pmatrix} \frac{A_{\pi\pi}}{1-iq_\pi A_{\pi\pi}} & \frac{A_{\eta\pi}}{1-iq_\eta A_{\eta\eta}} \\ \frac{A_{\pi\eta}}{1-iq_\eta A_{\eta\eta}} & \frac{A_{\eta\eta}}{1-iq_\eta A_{\eta\eta}} \end{pmatrix}, \tag{1}$$

where $q_{\pi,\eta}$ are the center-of-mass momenta of the two mesons in the two channels π, η. The channel scattering lengths A_{ij} are expressed in terms of the K-matrix elements, via the solution of $T = K + iKqT$,

$$A_{\pi\pi} = K_{\pi\pi} + iK^2_{\pi\eta} q_\eta/(1 - iq_\eta K_{\eta\eta}), \quad A_{\eta\pi} = K_{\eta\pi}/(1 - iq_\pi K_{\pi\pi})$$

$$A_{\eta\eta} = K_{\eta\eta} + iK^2_{\eta\pi} q_\pi/(1 - iq_\pi K_{\pi\pi}). \tag{2}$$

These equations form a basis in which to describe two channel scattering in terms of the

parameters of the K-matrix. These K-matrices must account for several observed features of the experimental data – in particular:

a) The S-wave πN resonances $S(1535)$ and $S(1650)$. The effect of these is inserted as poles at $E = E_0$ and E_1, which are treated as free parameters. However, their values are expected to be near 1535MeV and 1650MeV. Differences arise, since $E_{0,1}$ are renormalised by the presence of two background terms $K_{\pi\eta}$ and $K_{\eta\eta}$, which describe other forms of the interactions and channel couplings not included explicitly.

b) Experimentally, the η does not appear to couple to the $S(1650)$ resonance, and so this coupling is not included in the model.

c) There is a small correction for inelasticities of the $S(1535)$ and $S(1650)$ due to couplings to the two-pion nucleon channel. This is treated in an " optical potential manner", which means the introduction of the two-pion channel K matrix and its subsequent elimination. This leads to a complex correction to the two channel K matrix. For example, the $S(1535)$ has a coupling to the three-body channel described by a singular K-matrix $K_{3,3} = \frac{\gamma_3\gamma_3}{E_0-E}$ and its coupling to the two-body channels by $K_{3,i} = \frac{\gamma_3\gamma_i}{E_0-E}$, where $i = \pi, \eta$. In the three body-channel, there is a relative momentum equivalent to the above q_i. This is a three-body phase space element q_3. It may be included together with the coupling parameter γ_3 into a small contribution to the width of the $S(1535)$ width $\Gamma_{\pi,\pi}/2 = \gamma_3 q_3 \gamma_3$. Only this combination of the two-pion parameters enters the correction to the K matrix of the two channel problem. In principle, it should be proportional to the three body phase space, and this energy dependence has been accounted for. Now, the correction to the basic two channel K matrix, which stems from the three body channel is readily obtained to be

$$\delta K^0_{i,j} = i\frac{K_{i,3}q_3K_{3,j}}{1-iq_3K_{3,3}}. \tag{3}$$

A similar procedure is applied to describe the slightly higher inelasticity of the $S(1650)$ resonance.

These features are included in the K-matrices as follows.

$$K_{\pi\pi} \to \tfrac{\gamma_\pi(0)}{E_0-E} + \tfrac{\gamma_\pi(1)}{E_1-E} + i\tfrac{K_{\pi 3}q_3K_{3\pi}}{1-iq_3K_{33}} \ , \quad K_{\pi\eta} \to K_{\pi\eta} + \tfrac{\sqrt{\gamma_\pi(0)\gamma_\eta}}{E_0-E} + i\tfrac{K_{\pi 3}q_3K_{3\eta}}{1-iq_3K_{33}},$$

$$K_{\eta\eta} \to K_{\eta\eta} + \frac{\gamma_\eta}{E_0-E} + i\frac{K_{\eta 3}q_3K_{3\eta}}{1-iq_3K_{33}}, \tag{4}$$

where $\quad K_{33} = \tfrac{\gamma_3(0)}{E_0-E} + \tfrac{\gamma_3(1)}{E_1-E} \ , \qquad K_{\pi 3} = \tfrac{\sqrt{\gamma_\pi(0)\gamma_3(0)}}{E_0-E} + \tfrac{\sqrt{\gamma_\pi(1)\gamma_3(1)}}{E_1-E},$

$$K_{\eta 3} = \frac{\sqrt{\gamma_\eta\gamma_3(0)}}{E_0-E}. \tag{5}$$

In the above model, there are 10 parameters that are determined by a Minuit fit to 110 pieces of data – 23 are πN amplitudes (real and imaginary) [8], 11 are $\pi N \to \eta N$ cross-sections[$\sigma(\pi\eta)$] [9] and 53 are $\gamma N \to \eta N$ crossections [$\sigma(\gamma\eta)$] [10]. In practice, the actual cross section data was used in a reduced form, from which threshold factors have been removed – namely:

$$\sigma(\pi\eta)_r = \sigma(\pi\eta)\frac{q_\pi}{q_\eta} \quad \text{and} \quad \tau(\gamma\eta)_r = \sqrt{\sigma(\gamma\eta)\frac{E_\gamma}{4\pi q_\eta}}. \tag{6}$$

The values of $\tau(\gamma\eta)_r$ given in Ref. [10] are used here directly, even though the mass of the η there is 547.12MeV, compared with the present value of 547.45MeV in Ref. [12]. Such small differences are unimportant here, since the main threshold effect is removed

by considering the combination σ/q_η. In terms of the scattering amplitudes (T) of eq.(1), the corresponding model expressions are:

$$\sigma(\pi\eta)_r = 4\pi\left[(Re\ T_{\pi\eta})^2 + (Im\ T_{\pi\eta})^2\right]\frac{2q_\pi}{3q_\eta} \quad \text{and} \quad \tau(\gamma\eta)_r = A(Phot)\sqrt{(Re\ T_{\eta\eta})^2 + (Im\ T_{\eta\eta})^2},$$

where $A(Phot)$ is a normalisation parameter that simulates the actual production amplitude. This parameter is assumed to be energy independent and is treated as a free parameter in the Minuit minimization. The resulting fit had a χ^2 of 0.83/dof and the outcome is seen in Fig. 1. Since it is not clear that the four sets of data in Fig. 1 have equal weight, it is of interest to also look at the separate χ^2/dpt – A) 0.73 B) 0.75 C) 0.94 and D) 0.60. This shows that, indeed, good fits are achieved in all four sets of data and that the overall χ^2/dof is not dominated by any particular set.

From Table 1 it is seen that those parameters that can be compared with numbers in the Particle Data Tables [12] fall into three classes:

a) $\Gamma(Total)$, $\eta(br)$, $\pi(br)$, $\Gamma(Total, 1)$ and $\pi(br, 1)$ can be compared directly and are seen to be consistent with the experimental uncertainties. The relationship between the above Γ's and the γ's is determined by the T matrix, which – close to the resonance – should be of a Breit-Wigner form with an energy dependent width. This relates the channel parameters γ to the total width Γ , with elasticities and the channel momenta calculated at the resonance energy $q(PDT)$. Thus, for example $\gamma_\pi = 0.5\pi(br)\Gamma/q(PDT)$.

b) E_0 and E_1 are the positions of the bare poles in the K-matrices. As mentioned earlier, these get slightly renormalised in going from K-matrices to T-matrices to give the numbers in the Particle Data Tables [12].

c) The seven parameters in a,b) are essentially obtained by fine-tuning the corresponding experimental numbers – as is seen by the close agreement between the two. However, the remaining three parameters $K_{\eta\eta}$, $K_{\pi\eta}$ and $A(Phot)$ are completely free. In principle, the first two could be related to some more fundamental model based on some underlying lagrangian as in Refs. [13]. The third parameter could also be calculated, if a mechanism for η photoproduction were used.

The values of the branching ratios $\eta(br)$, $\pi(br)$ for the $S(1535)$ resonance also give a prediction for the two-pion ratio to be $1 - \eta(br) - \pi(br) = 0.038$ – a number in line with experimental estimates of 0.05–0.20.

The errors on a, r_0 and s were obtained by repeating the calculation for a random selection of the nine parameters defining the K-matrices of eq.(1). This selection was chosen to ensure the distribution of each parameter was a gaussian centered on the values in Table I and with the same standard deviation. Several tests were made to determine the dependence of these errors on the number of runs and on the size of the region each side of the gaussian maximum over which the random points were chosen. The errors shown are for 1000 runs using regions that were 3 standard deviations.

The negative sign of the effective range is expected, since it arises quite naturally due to the proximity of the $S(1535)$. For the single η channel case dominated by the resonance, one would have $r_0 q_\eta^2/2 = (E_{threshold} - E)/\gamma_\eta$. This is a fairly large negative effective range of about –3 fm. The presence of other channels and background terms reduce it to about half of this value. The shape parameter appears to be small. In fact the imaginary part is consistent with zero.

In Table 2 a comparison is made with earlier determinations of the scattering length. There it is seen that the present result supports, in particular, the estimates of Refs. [4] and [18]. It is difficult to compare with the other references, since they do not give any error estimates.

Fig. 2 shows that, within 30MeV of the η threshold, the effective range expansion is very good. For a parametrization up to 100MeV from the threshold, the effect of the shape parameter(s) plays an increasingly important role. Also it is seen that the effective range must be included, if the ηN scattering is needed 10–20MeV away from the η threshold at 1485.7MeV. Such excursions from the threshold are needed, for example, when extrapolating below the threshold in η-bound state situations. In the present case, the threshold value of the ηN amplitude (0.75+i0.27) becomes 0.49+i0.10 fm at 1468.4MeV and 0.51+i0.51 fm at 1500.0MeV. Such differences could be crucial in discussions concerning the existence, or not, of η-bound states in few nucleon systems.

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TABLES

TABLE I. The optimised parameters from Minuit defining the K-matrices and the corresponding values from the Particle Data Tables (PDT) [12].

	$K_{\eta\eta}$	$K_{\pi\eta}$	E_0(MeV)	E_1(MeV)	$\Gamma(Total)$(MeV)
Minuit	0.177(33)	0.022(13)	1541.0(1.6)	1681.6(1.6)	148.2(8.1)
PDT	–	–	1535(20)	1650(30)	150(50)
	$\eta(br)$	$\pi(br)$	$\Gamma(Total,1)$(MeV)	$\pi(br,1)$	$A(Phot)$
Minuit	0.568(11)	0.394(9)	167.9(9.4)	0.735(11)	19.74(36)
PDT	0.30–0.55	0.35–0.50	145–190	0.55–0.90	–

TABLE II. Results compared with earlier works. The numbers in [...] are the values of a and r_0, when the exact scattering amplitudes are fitted with $s = 0$.

Reference	Scattering Length(fm)
Bhalerao and Liu [14]	0.27+i0.22
	0.28+i0.19
Benhold and Tanabe [15]	0.25+i0.16
Arima, Shimizu and Yazaki [16]	0.980+i0.37
Švarc, Batinic and Slaus [17]	0.886+i0.274
Wilkin [4]	0.55(20)+i0.30
Sauermann et al. [13]	0.51+i0.21
Abaev and Nefkens [18]	0.621(40)+i0.306(34)
This paper	
Scattering length(a)	0.751(43)+i0.274(28)
	[0.751(43)+i0.274(28)]
Effective range(r_0)	–1.496(134)–i0.237(37)
	[–1.497(134)–i0.237(38)]
Shape parameter(s)	–0.102(15)–i0.008(10)

FIGURES

FIG. 1. The K-matrix fit to experimental data as a function of the center-of-mass energy Ecm: A) The $\pi N \to \eta N$ data of Ref. [9] – the reduced cross-section in mb containing the factor q_π/q_η, B) $\tau(\gamma\eta)_r$ the reduced cross-section of Ref. [10] in units of $10^{-3}/m_{\pi^+}$, C) The real part of the πN amplitudes (q_π Re T) [8], D) The imaginary part of the πN amplitudes (q_π Im T) [8].

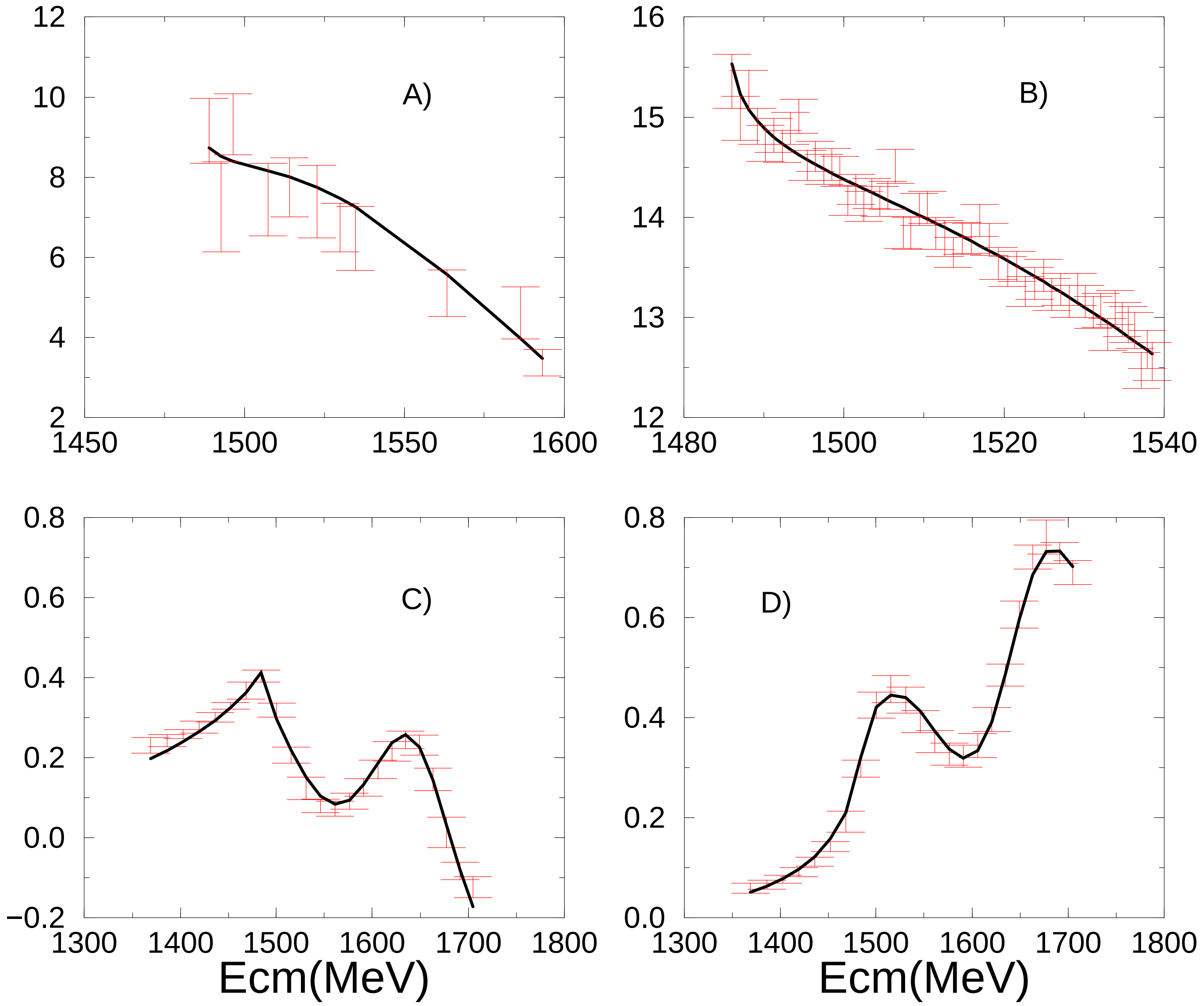

FIG. 2. The quality of the effective range expression versus the exact values. The solid line shows the exact results, the dashed line the effective range expansion with the values of $a,\ r_0,\ s$ from table 2 and the dotted line the effective range expansion with only $a,\ r_0$. A) shows the real parts and B) the imaginary parts. All amplitudes are in fm.